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A Bundle Pricing Approach for Mobile Telecommunication Services: Method and Data Analysis

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Abstract
The bundling of goods/services is a technique many firms use to influence product demand, generate higher revenues, and enhance consumer surplus. In the telecommunications industry, offering incentive bundles of different mobile phone services is an effective technique to reach such goals in a competitive market. This paper presents a bundle pricing approach for mobile services, which determines the optimal content of service bundles in terms of the type and number of services offered to different customer segments. The proposed model aims to maximize the total firm's revenue and total consumer surplus, as the main mobile service operator's objectives. The model recognizes differential pricing as a useful tool in revenue management. First, an efficient segmentation of customers in terms of their taste and willingness to pay for different mobile services is conducted using the k-means clustering technique. Next, to handle customer buying behavior, the customer reservation price is considered based on the customers' arrival rates and their statistical distribution. Finally, the bundles' content and prices are optimized considering the type and number of services offered to different segments. Our computational experiments using sample data show the effectiveness of the proposed model toward the improvement of revenue as well as consumer surplus.

Keywords: Bundling, Telecommunication industry, Differential pricing, Consumer surplus, k–means clustering.

JEL classification: M10, M21, M30.

1. Introduction

Firms must make an essential decision to price goods/services to preserve potential customers and market share while maintaining profits. In many industries, however, production can only sometimes be balanced with demand, which could lead to lost revenue potential (van Ryzin and Talluri, 2005). The bundling of physical or non-physical goods/services is among the ways firms influence customer demand. This technique typically puts together products that are mainly replaceable by each other. At the same time, their demand distribution functions have an inverse relationship and are set at a lower price than the summation of their prices. Therefore, customers may be encouraged to buy more items, which would lead to the firm's strong position in the market and improved profitability.

In the literature, there are various definitions of bundling or bundle selling. Adams & Yellen (1976) considered bundling as selling products in a unit package. Guiltinan (1987) defined bundling as selling a bundle of two or more products at a specific price. Stremersch & Tellis (2002) distinguished between price bundling and product bundling. Price bundling was defined as selling products in a non-physical bundle with a specific discount. In contrast, product bundling was defined as selling two or more products in a physical bundle at a single price. A recent study on emerging trends identified bundling as a strategy where two or more products, physical or non-physical, are offered together at a discounted price. (Rao *et al.*, 2018). In this paper, a different characterization of bundling is considered, where the frequency of goods/services is considered as well. Hence, we define bundling as putting together two or more physical or non-physical goods/services, which may differ in number but are the same in price.

Introducing a bundle can alter the range of choices available to consumers, thereby, given the context, impacting their purchase behavior (Yin, Jiang and Zhou, 2023). Bundling has several potential benefits for both sellers and buyers (customers), including the improvement in income and profit. For sellers, it could provide benefits such as receiving the total price of each bundle at the beginning of the sales period, helping to introduce new products to the market, reducing the transaction cost, exploiting the economies of scale, extending the economies of scope, monitoring sales and inventory more efficiently, reducing the intensity of competition in the market, and achieving partial monopoly power. On the other hand, customers could benefit from the convenience of payment (one bill for several products), enhanced surplus, and purchase discounts (Chopra and Meindl, 2007; Derdenger and Kumar, 2013). Because of the benefits mentioned above, bundle selling has traditionally been applied in many businesses. A well-known example is the software bundle of Microsoft Office, which contains several applications. Tour services (e.g., tickets and accommodation), food packages, and even data plans offered by mobile carriers are other examples of bundles commonly used in practice.

While bundling can be practical, sellers must decide which strategy would be more efficient and how to put products into a bundle and price them. Several factors would add to the complexity of such decisions, such as the variety of products, the broader aspects of market competition, and the importance of paying attention to the intelligent behavior of customers in the market (Venkatesh and Mahajan, 2009). For example, customer behavior may include the willingness to pay for organic and/or fairtrade products (Nicolae & Roșca, 2022; Pracejus and others, 2022). Moreover, as more expensive products are generally more profitable and have lower demand than less expensive alternatives, sellers, including service-providing firms, always look for optimal ways to supply cheap and expensive products together according to their available capacity (Yang and Ng, 2010).

Bundling has been used in the mobile telecommunication industry for a considerable duration, and the corresponding decisions are relevant and critical (Sridhar and Sridhar, 2019). From a mobile telecommunication service provider's perspective, an appropriate service bundling technique could benefit the firm and its customers significantly.

The general research question we aim to address in this paper is how to bundle mobile services to benefit the firm and its customers. More specifically, we present a three-phase methodology to optimize mobile service bundles and their prices, in terms of revenue and consumer surplus, given the type and number of services. While the literature on the narrower domain of mobile telecommunication service bundling consists of studies on customer perceived value (Klein and Jakopin, 2014), customers' present and future choices (Üner, Güven and Cavusgil, 2015), and customer preferences for service improvements (Dagli and Jenkins, 2016), none directly addresses our specific problem and modeling approach. The proposed model is consistent with heterogeneous customer tastes and aims to maximize the total firm's revenue and total consumer surplus as the leading mobile service provider's objectives. Given the importance of price differentiation in revenue management, first, an effective segmentation of mobile customers in terms of their tastes and willingness to use services is carried out. A *k*-means clustering approach is used to group customer purchase behavior. Customer buying behavior is then described using reservation price, and appropriate bundles are prepared to increase willingness to use services. The goal is to maximize total revenue and consumer surplus separately. Computational experiments are provided to investigate performance.

The rest of the paper is organized as follows. Section 2 provides a review of the most relevant literature. In section 3, the proposed mobile bundle pricing methodology is presented. Sample data and numerical analyses are discussed in section 4. Finally, conclusions and future research directions are outlined in Section 5.

2. Literature Review

The most relevant literature is discussed in the following paragraphs. We start this section with the research works generally related to bundling and continue our discussion with the literature more specific to the bundling of information goods and mobile services.

Selling goods and services as a bundle has traditionally been used in many businesses to generate higher revenues. However, the first scientific research in this area was conducted in the early 1960s: Stigler (1963), for the first time, argued how a customer's willingness to buy a bundle of two negatively interdependent goods could increase the seller's profit. Adams & Yellen (1976) considered a monopolist firm selling two different products in a specific bundle; they determined optimal sales strategies under the assumptions of technology (the marginal cost of supplying products in the bundle is the sum of its component costs), indivisibility (the marginal utility from the second unit of product is equal to zero), and independence (the customer's willingness to pay or customer's reservation price for a bundle is equal to the sum of its items' reservation prices). Schmalensee (1984) improved the model proposed by Adams & Yellen (1976) through a bundling model for a monopolist who sells two types of products, where reservation prices follow a two-variable Gaussian distribution.

Hanson & Martin (1990) were the first researchers to present a method for calculating optimal bundle prices for multi-product firms. Salinger (1995) analyzed how bundling can affect a firm's profitability by comparing bundle demand and contents. Brooks et al., (2001) optimized a bundle pricing model for variable and unpredictable customer demands. Hitt & Chen (2005) discussed customized bundling, where consumers can choose a bundle from many products to attain its price discount. The existing literature has also studied competition and investigation of duopoly markets (Vaubourg, 2006; Thanassoulis, 2007). Eckalbar (2010) studied a monopolist selling bundles of two different products to a group of customers that have uniformly distributed reservation prices. For a more detailed review of relevant works on optimal bundling, we invite the reader to consult Fuerderer, Herrmann, & Wuebker (2013), Vamosiu (2018), and Rao et al. (2018).

In recent years, the development of new technologies, e-commerce, and the entrance of new competitors to the market have led to different applications of bundling. The bundling of information goods with a low marginal cost such as mobile telecommunications services is one of them. In this domain, Bakos & Brynjolfsson (2000) investigated bundling strategies for a multi-product monopolist firm supplying information goods; they found that bundling lots of information goods could be surprisingly profitable for the firm. Wu & Anandalingam (2002) presented a model to optimize the number of software bundles and their prices toward the design of a new market for information goods. Also, Venkatesh & Chatterjee (2006), Shiller & Waldfogel (2011), and Crawford & Yurukoglu (2012) studied the optimal bundling and pricing strategies of journals (printed and electronic), music tracks (album), and television channels, respectively. The research works conducted by Hiller (2017) and Banciu, Ødegaard, & Stanciu (2022) can be mentioned as a couple of more recent studies on information goods bundling.

Investigating mobile telecommunication services, as information goods, is an attractive area in the bundling and pricing literature. In this regard, Juha & Minna (1970) studied the properties of mobile service markets that use bundling strategies in Finland and the

Scandinavian market. Bouwman, Haaker, & De Vos (2007) investigated the bundles of mobile services that were more attractive to customers, and using continuous analysis, they evaluated the best combinations of services and price levels. Yang & Ng (2010) defined a mixed price bundling problem in the context of mobile wireless telecommunication, where the bundle prices were determined in such a way that the total seller's profit is maximized. Bundling in telecommunication services has also been studied as a measure for customer churn reduction (Prince and Greenstein, 2014). Klein and Jakopin (2014) investigated how users perceive the utility of mobile service bundles and their willingness to pay for such bundles. Through empirical research conducted in the Turkish market, Üner, Güven and Cavusgil (2015) analyzed consumers' present service bundle choices and their future intentions. Finally, Dagli and Jenkins (2016) utilized a choice experiment to assess consumers' willingness to pay for enhancements in mobile services, with a specific focus on 4G upgrades and roaming services.

Based on our review, the bundling technique is increasingly growing in selling information goods, e.g., mobile telecommunication services. Given the relevance and importance of this topic, this paper presents a methodology for bundle pricing in the widespread and highly competitive mobile telecommunication markets, where various services are offered. Although the existing literature covers various topic dimensions, none directly pertains to our specific problem and chosen modeling approach. In the next section, we introduce our proposed bundling method.

3. Method

Effective segmentation of a mobile service operator's customers in terms of their tastes and willingness to use different services is considered the first step toward a proper bundling method (Phase I). While it is important to consider customer buying behavior, its comprehensive analysis requires extensive effort and time, e.g., for data acquisition through direct ways or experimental designs; therefore, as an alternative technique, probability distributions are used to describe customer buying behavior in each segment (Phase II). Then, a mathematical optimization model is solved to maximize the total firm's revenue and total consumer surplus (Phase III).

3.1. Assumptions

The model assumptions are listed below:

1. The mobile customer community has diverse tastes and varying willingness to pay for each service. This taste heterogeneity is among the most critical elements of business price management.
2. Customer demand in each period is similar to that of previous periods. This assumption helps us decide current consumption according to prior periods' demand.
3. The proposed bundles are perishable. It means that when a period is expired, the bundles cannot be transferred to the next period.
4. Each bundle is active only for one consumer, and sharing one bundle between two or more customers is impossible.
5. For each customer in each segment, there is only one opportunity to use services in the form of a bundle and its discount. It means that every customer will meet his/her extra needs only by individual buying.
6. The bundles offered to each segment are dedicated only to that segment, and other segments cannot access them.
7. Since mobile telecommunication services are placed among the information goods, the marginal cost to produce them is low and assumed to be zero. Naturally, the mobile service operator is faced with multiple fixed costs such as setup, installation, and

maintenance services costs. However, these types of costs are not considered in this study.

8. Customer demand follows a normal distribution.
9. The distribution of reservation prices for each service in each segment has a normal distribution with unknown parameters. It is necessary to explain that this assumption has a strong foundation; in most studies, asymmetric Gaussian distribution (generally having the right skewness) was obtained as an empirical distribution for reservation prices based on actual data (Schmalensee, 1984). Moreover, various types of asymmetric Gaussian distribution with different parameters can easily be converted to a standard normal distribution. According to Schmalensee (1984), the frequent use of Gaussian distribution in social sciences to represent customer tastes is a good reason for using this distribution to describe customers' reservation prices.
10. Customers are connected to the mobile network according to a Poisson process.
11. Different units of service have independent value from the customer's perspective. This assumption is essential to consider since ignoring it in some cases (e.g., consumer goods with a long life or some goods with relatively high appearance importance) could lead to bad decisions in price management. For instance, consider a bundle of two similar suits; the reservation price of the bundle increases less than expected because people are rarely willing to buy two similar suits. However, the abovementioned assumptions are valid for consumer goods with a short life or low appearance importance, such as food items.
12. The customer's reservation prices can be added together. In other words, each bundle's total customer reservation price can be calculated by the sum of its components' reservation price.
13. The mobile service operator has monopoly power in the market. It is assumed true as long as there is no significant difference between the price offered by the operator and its competitors. Hence, customers are unwilling to receive services from them due to the additional cost of subscribing to other operators.
14. Customers in each segment only purchase a bundle that would lead to a surplus and need at least all units of one of the service types in the offered bundle.

3.2. High-level Methodology

The proposed approach includes three distinct phases as follows (Figure 1):

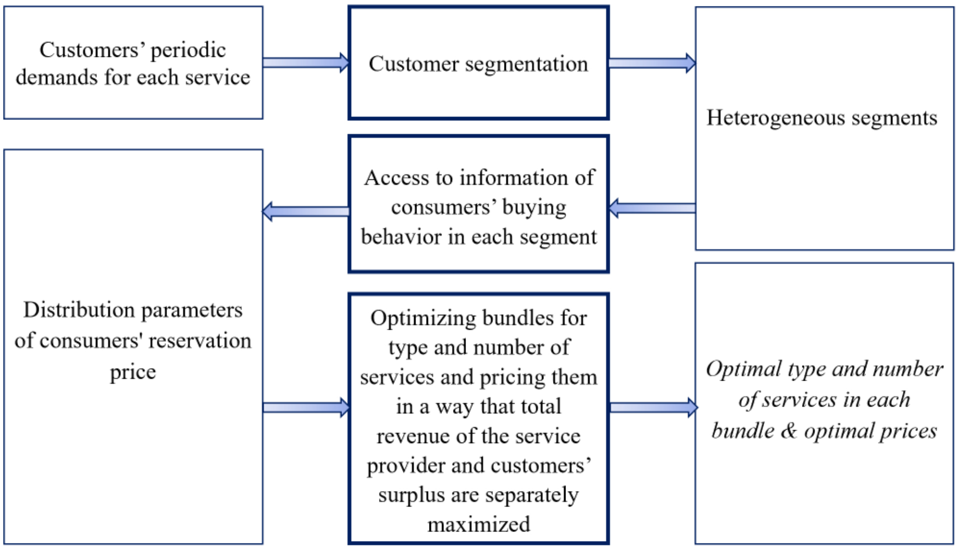

Figure 1: The proposed modeling approach for bundle pricing of mobile services

- *Phase I-* Customer segmentation based on customer consumption data.
- *Phase II-* Determining customer buying behavior in each segment and estimating the distribution parameters of customer's willingness to pay.
- *Phase III-* Optimizing the proposed bundles considering the type and the number of their services and pricing them for each segment such that the mobile service operator's total revenue or consumer surplus is maximized.

To find more information on the mathematical details of the abovementioned phases, see Appendices A (notations), B (clustering), C (reservation price), and D (optimization problem).

4. Data Analysis and Discussion

In this section, we illustrate the application of our proposed methodology using a sample data set. The sample data set contains local voice calls and short message services (SMSs) of a random sample of 2,000 active mobile customers in two monthly billing periods (collected from Hamrah-e-Avval, a major telecommunication service provider in Iran); the sample was randomly drawn from 300,000 active customers who held permanent SIM cards of the company. While only these two service types, i.e., voice call and SMS, are considered here for illustration purposes, the proposed methodology is capable of including other services such as multimedia messages or mobile internet as well.

To evaluate the model's performance, the dataset is analyzed. The customer community is segmented using *k*-means clustering, a simple yet effective algorithm for partitioning and segmenting customers (Kansal *et al.*, 2018). Results and discussion focus on total revenue and consumer surplus.

4.1. Customer Segmentation

Data clustering was performed on a sample of 2,000 active mobile customers for 20 distinct clusters using *k*-mean clustering; while we did not explore other segmentation procedures, we invite the readers to consult Leisch, Dolnicar and Grün (2018) for a comprehensive review of segmentation methods. The clustering criteria were customer demand for two services, i.e., local voice calls per minute and SMSs (the data was standardized to make each clustering criterion a free scale).

Table 1. Clustering results

Cluster number	Number of customers	Average voice call (per minute)	Variance of voice calls	Average demand for SMS	Variance of demand for SMS	Upper bound of demand for voice calls	Upper bound of demand for SMS
1	2	5,811	300.52	814	82.02	6,023	872
2	2	201	41.72	4,203	304.76	230	4,418
3	7	2,831	336.70	751	242.09	3,326	1,215
4	193	734	126.22	69	68.83	1,009	252
5	5	3,894	243.47	200	215.90	4,169	553
6	86	1,222	147.42	165	117.38	1,550	475
7	18	1,096	260.36	1,469	227.73	1,670	1,806
8	58	200	127.30	592	123.33	436	859
9	8	1,838	253.32	953	328.84	2,234	1,383
10	3	593	231.56	3,152	193.47	860	3,315
11	31	448	237.96	1,095	168.14	929	1,458
12	846	97	64.84	29	35.91	237	164
13	396	372	89.63	47	48.31	556	207
14	2	898	7.78	3,725	101.82	903	3,797
15	65	742	192.48	445	138.94	1,245	763

16	12	2,914	307.73	115	78.09	3,315	263
17	3	997	469.16	2,177	156.84	1,360	2,308
18	2	4,893	167.58	172	36.77	5,011	198
19	223	214	119.19	237	70.26	527	406
20	38	1,924	274.80	104	106.06	2,415	399

After 58 iterations, clustering reached the result (Table 1). According to our experiments, 20 clusters allowed for more detailed analysis, and none of the clusters consisted of just a single member. The latter implies the absence of an outlier in the data. Figure 2 presents the scatter plot of voice calls and SMSs in the obtained clusters by different colors.

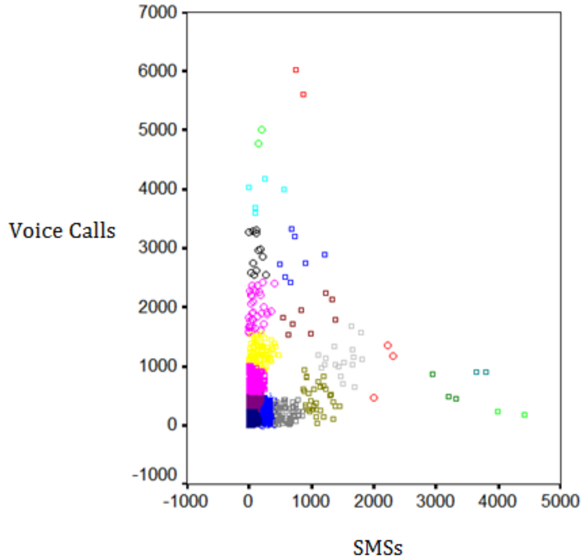

Figure 2: The scatter plot of voice calls and SMSs in the obtained clusters from the first phase

Following the completion of the first phase, probability distributions were used to evaluate the customer’s willingness to pay (Audzeyeva, Summers and Schenk-Hoppé, 2012) in the second phase, where their reservation prices were determined. Finally, the optimal bundle configuration and pricing were determined in the third phase, using Simulated Annealing (SA), a metaheuristic solution algorithm (Talbi, 2009). Two scenarios are explored below.

4.2. First scenario: Bundling to maximize total revenue of mobile service operator

The first scenario is to maximize the mobile service operator's total revenue. Single SMS and one-minute voice call cost 134 and 447.5 Iranian Rial (IRR), respectively. Table 2 shows the detailed results of the first scenario. The optimal total revenue of 506,500,682 IRR is reached in 17.4626 seconds.

Table 2. Computational results (first scenario)

Cluster number	**Reservation price for offered bundle**	**Amount of voice calls per minute**	**Number of SMS**	**Bundle prices (IRR)**	**Surplus for cluster**
1	2,795,083	6,000	800	2,750,000	90,166
2	680,779	200	4,400	640,000	81,558
3	1,533,357	3,300	400	1,490,000	303,501
4	461,243	1,000	100	430,000	2,093,309
5	1,901,747	4,100	500	1,860,000	83,495
6	457,517	900	400	430,000	2,366,545
7	904,923	1,600	1,400	850,000	604,157
8	230,631	400	400	210,000	1,196,632

9	1,147,153	2,200	1,300	1,100,000	141,460
10	801,358	800	3,300	760,000	82,717
11	520,391	900	900	490,000	881,355
12	87,253	200	0	80,000	6,136,719
13	224,098	500	0	210,000	5,582,846
14	886,055	900	3,600	840,000	92,111
15	563,751	1,200	200	530,000	2,193,855
16	1,478,688	3,300	100	1,410,000	480,817
17	851,492	1,300	2,000	800,000	154,478
18	2,239,371	5,000	0	2,120,000	238,743
19	233,504	500	100	220,000	3,011,609
20	1,055,232	2300	300	1,010,000	226,164

The second column of Table 2 includes the optimal value of the average customer's willingness to pay or the customer's reservation price for the offered bundle to each cluster. This value for all clusters is higher than the price of the offered bundle, which implies that the offered price to customers is reasonable. The third to fifth columns correspond to optimal values of the number of voice calls per minute, number of SMSs, and price of each bundle as the optimization model variables. For example, if the mobile service operator presents a bundle of 800 text messages and 6,000 minutes of voice calls with the price of 2,750,000 IRR to the first cluster, it will lead to maximum revenue. The sixth column of Table 2, accounting for those customers who are willing to buy the offered bundles, shows the cluster-specific consumer surplus amounts.

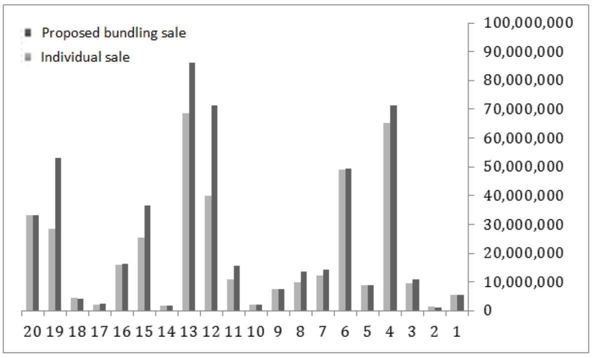

Figure 3: Revenue comparison between the proposed bundling sale and individual sale (first scenario)

Figure 3 compares the results of the proposed bundling methodology vs. individual sales by dark and light columns, respectively. The generated revenues from the proposed method in most of the clusters are more than those of individual sales. Based on the results, the total revenue of the mobile service operator using the proposed method is 506,500,682 IRR, which shows significant improvement over that of individual sales (i.e., 401,662,223 IRR). Moreover, the total consumer surplus is obtained at 26,042,237 IRR. Our observations are consistent with previously conducted studies such as Derdenger and Kumar (2013), as they show how proper bundling could lead to both improved firm profitability and enhanced consumer surplus.

4.3. Second scenario: Bundling to maximize total consumer surplus

In addition to generating revenue through bundle selling, other objectives can be important for a firm. One of these objectives is maximizing consumer surplus in all or some clusters. For example, a new mobile service operator may want to increase its share in the market or keep its loyal customers. In such cases, the operator may accept losing some potential profit and instead aim for increasing surplus for such customers. Moreover, the operator may want to attract customers who show a desire for a newly launched service and are a good market for it. Therefore, the operator maximizes the total consumer surplus as the second objective of the pricing strategy. In this regard, an important issue is optimizing consumer surplus, which requires the lowest possible amount of feasible price for each proposed bundle. We assume that the maximum amount of discount for each bundle is 15% of the sum of contents values.

The model maximizes total consumer surplus with equal cluster weights. Table 3 shows the second scenario results: 57,037,663 IRR in 16.8496 seconds.

Table 3. Computational results (second scenario)

Cluster number	Reservation price for offered bundle	Amount of voice calls per minute	Number of SMS	Bundle prices (per IRR)	Surplus for cluster
1	2,795,083	6,000	800	2,380,000	830,166
2	680,779	200	4,400	580,000	201,558
3	1,238,033	2,400	1,200	1,050,000	1,316,237
4	284,140	600	100	240,000	7,768,718
5	1,633,918	3,500	500	1,390,000	1,219,595
6	489,582	1,000	300	420,000	5,775,342
7	594,744	900	1,400	510,000	1,525,408
8	230,631	400	400	200,000	1,776,632
9	838,821	1,500	1,300	720,000	950,574
10	787,997	800	3,200	670,000	353,993
11	390,501	600	900	340,000	1,515,051
12	87,253	200	0	80,000	6,136,719
13	224,098	500	0	200,000	9,542,846
14	899,432	900	3,700	770,000	258,865
15	563,751	1,200	200	480,000	5,443,855
16	1,478,688	3,300	0	1,260,000	2,624,258
17	851,492	1,300	2,000	730,000	364,478
18	2,253,222	5,000	100	1,920,000	666,445
19	233,504	500	100	210,000	5,241,609
20	702,771	1,500	300	610,000	3,525,314

In this scenario, the firm's total revenue increased significantly from IRR 401,661,781 in individual sales to IRR 463,365,737 in bundle selling. The generated revenue in the second scenario is lower compared to the first scenario, which is expected as the revenue is not being maximized in the second scenario. Despite our expectation of losing revenue in bundle selling, the revenue has increased. It is consistent with the observations in the first scenario and shows how bundle selling could be a win-win strategy whether revenue or consumer surplus is maximized.

Figure 4 compares revenue obtained from the proposed bundling procedure and individual sales. While in most clusters, the generated revenue through individual sales is more than the bundling sales, significant growth in total revenue is evident. Again, our observations show how bundling could be an effective strategy for firm profitability as well as consumer surplus improvements. While we do not illustrate additional scenarios, one could do so by combining the revenue and consumer surplus components with different weights and solve the resulting problem.

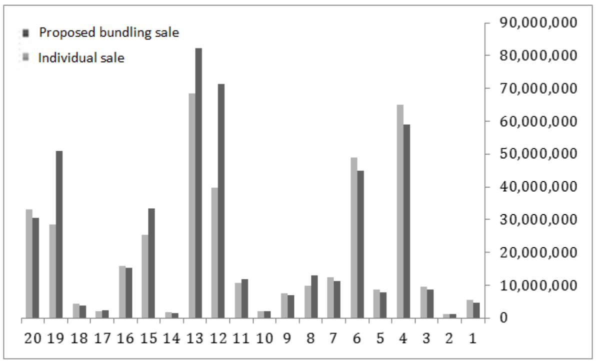

Figure 4: Revenue comparison between the proposed bundling sale and individual sale (second scenario)

5. Conclusion

Product bundling is a common practice in the telecommunication industry, where different mobile phone services are bundled and offered to various customer segments. This approach can influence product demand, generate higher revenues, and improve consumer surplus. This paper presents a bundle pricing approach for mobile services to determine the optimal content of service bundles offered to customers.

Based on our literature review, bundling is an increasingly growing and effective technique for selling information goods, e.g., mobile telecommunication services. Given the relevance and importance of this topic, we focused on bundle pricing in the mobile telecommunication markets, where there is a variety of services provided for customers.

We included three distinct phases in our proposed modeling approach. In the first phase, customers were segmented based on their taste and their willingness to pay for different mobile services; such segmentation was conducted using a simple *k*-means clustering technique. In the second phase, to account for customer buying behavior, customer reservation prices were determined given the customers' arrival rates and their statistical distribution. Then, the content of offered bundles and their prices were optimized in the third phase considering the type and number of services offered to different segments.

To test the effectiveness of the proposed model toward the improvement of revenue as well as consumer surplus, we conducted computational experiments on sample data. Two different scenarios were considered. In the first scenario, the model objective was maximizing the total revenue of the mobile service operator. Then, maximizing the total consumer surplus was investigated as a second scenario. To this end, the proposed model was applied to the sample data according to the three phases of the model. Consistent with previous studies, our findings demonstrated the potential effectiveness of bundling as a strategy to enhance firm profitability while also benefiting consumer surplus.

As the study undertaken in this paper is associated with certain assumptions and limitations, it can be extended in different ways. For instance, customer willingness to pay could be investigated through continuous analysis and various econometric methods for different mobile services. Further, the customer community could be segmented using methods and algorithms other than *k*-means clustering, and factors such as age, gender, and occupation could be included in the segmentation. Using larger relevant data sets on customer demand and preferences could also lead to additional useful insights. Finally, other considerations such as bundling costs, the importance of service quality, and limitation on types of offered services could be incorporated into the model.

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Appendix A – Notations

The following notations (model parameters and decision variables) are used in the proposed methodology:

Parameters

TI: Length of the billing period or the amount of credit bundle per minute.

N_i: Total number of customers in cluster i $(i = 1, \dots, I)$.

PR_k: The real price of service k $(k = 1, \dots, K)$.

C_i: The cost of the offered bundle to cluster *i*.

τ_k: Time interval between two successive uses of service *k*.

λ_{ik}: Customer arrival rate to cluster *i* for using service *k*.

$W_{ik}^{m_i}(t)$: The number of service *k* which is bought by each customer of cluster *i* in time interval *t*.

$U_{ik}^{m_i}$: The amount of demand for service k requested by the customer m_i of cluster i in the billing period TI, which is normally distributed with parameters $(\mu_{ik}, \sigma_{ik}), m_i = 1,2, \dots, N_i$.

$R_{ik}(\tau_k)$: The average willingness to pay or reservation price of customers within cluster *i* for service *k* provided that the time interval between two successive consumptions of service *k* be equal to τ_k.

v_1: Rounding coefficient of offered bundle price to cluster *i*.

v_2: Rounding coefficient of the number of service *k* in the offered bundle to cluster *i*.

w_i: The weight of each cluster, based on its importance in terms of survival in the customer community.

γ_1: Coefficient of total revenue.

γ_2: Coefficient of total consumer surplus.

Decision Variables

n_{ik}: The number of service *k* in the offered bundle to cluster *i*.

p_i: Price of the offered bundle to cluster *i*.

S_i: Surplus created for customers of cluster *i* in the case of purchasing the offered bundle to the cluster.

RE_i: Customer’s reservation prices of cluster *i* for the offered bundle to this cluster.

$H_{ik}^{m_i}$: A binary variable which is one if the customer m_i of cluster *i* has a demand for service *k* more than what is included in the offered bundle to this cluster.

$X_i^{m_i}$: A binary variable which is one if the customer m_i of cluster *i* purchases the offered bundle.

Appendix B – Clustering

To segment customers, a *k*-means clustering algorithm is used, which contains five steps as follows:

1- An appropriate number of clusters is chosen.
2- For each cluster, a point is randomly selected as an initial guess for the cluster center.
3- All data are allocated to the clusters based on distance criteria.
4- The new center of each cluster is obtained by averaging the cluster members.

5- Steps 3 and 4 are repeated until all clusters are stable and there is no change in the clusters.

Let I show the number of clusters. Then, customer clustering can be performed by minimizing the following objective function:

$$Min\ Distance = \sum_{i=1}^{I} \sum_{Customer} \left\| u_{i.}^{Customer} - center_{i.} \right\|^2 \tag{1}$$

In the above equation, $\| \ \|$ is the distance criterion based on the Euclidean system. The $center_{i.}$ is the center of the i^{th} cluster and $u_{i.}^{\boldsymbol{Customer}} = u_{i1}^{\boldsymbol{Customer}}, \ldots, u_{iK}^{\boldsymbol{Customer}}$ is the normalized demand vector of each customer for each service during the *TI* period. It is a value in the range [0,1], which is obtained as below:

$$u_{ik}^{\boldsymbol{Customer}} = \frac{U_{ik}^{m_i} - min(u_{ik})}{max(u_{ik}) - min\ (u_{ik})} \tag{2}$$

In equation (2), $U_{ik}^{m_i}$ is the amount of demand for service *k* requested by the customer m_i of cluster *i* in the billing period TI.

Appendix C – Reservation Price

Customer reservation price is considered in relation to the time intervals between purchases, focusing on products with a short lifetime that can be bought multiple times within a planning horizon.

Let PR_k represents the price of service *k* proposed by the mobile operator in a situation of individual sale. Each customer of cluster *i* enters the system according to a Poisson process with an *average call rate per minute* λ_{ik} for buying service *k*. It is assumed that the users who enter the system would buy at least one service. Consider $W_{ik}^{m_i}(t)$ as the number of service *k* bought by each customer of cluster *i* at time interval *t*. The probability of selling *w* number of service *k* to a customer m_i of cluster *i* at price PR_k in time interval τ_k, can be calculated as follows:

$$p\left(W_{ik}^{m_i} = w | T \le t \le T + \tau_k\right) = \frac{e^{-\lambda_{ik}\tau_k} \times (\lambda_{ik}\tau_k)^w}{w!} \tag{3}$$

In the above equation, if τ_k approaches to *TI*, then $W_{ik}^{m_i}$ is equal to $U_{ik}^{m_i}$ as the amount of demand for service *k* requested by the customer m_i of cluster *i* in the billing period TI.

The customer arrival rate to the system can be used to calculate the probability of entering customers who have a reservation price greater than the current one, which could in turn help us estimate the distribution of customers' reservation prices. A customer's reservation price is considered to be equal to the maximum price that a customer is willing to pay for a given product. Based on this definition, when the product price in a given time interval is below the customer's reservation price, the customer tends to buy or use it in that time interval. Consequently, "higher reservation price than set price" and "not increasing the time interval between purchases" can be interpreted as two equivalent events. Therefore, the probability that customer m_i in time interval $T \le t \le T + \tau_k$ at least once attempts to use the service *k* at price PR_k, and the probability that the time interval time between two purchases at the price PR_k does not increase are the same:

$$p\left(W_{ik}^{m_i} \ge 1 | T \le t \le T + \tau_k\right) = p\left(R_{ik}^{m_i}(\tau_k) \ge PR_k\right) \tag{4}$$

Using equation (3), equation (4) can be rewritten as follows:

$$p\left(W_{ik}^{m_i} \ge 1 | T \le t \le T + \tau_k\right) = \sum_{w=1}^{\infty} \frac{e^{-\lambda_{ik}\tau_k} \times (\lambda_{ik}\tau_k)^w}{w!} = 1 - \frac{e^{-\lambda_{ik}\tau_k} \times (\lambda_{ik}\tau_k)^0}{0!}$$

So we have:

$$p\left(R_{ik}^{m_i}(\tau_k) \geq PR_k\right) = 1 - e^{-\lambda_{ik}\tau_k} \tag{5}$$

Equation (5) can be also derived in another way as follows. Assume variable T_{ik} shows the time interval between two consecutive uses of service k in cluster i, which follows an exponential distribution with parameter λ_{ik} namely:

$$T_{ik} \sim \exp(\lambda_{ik}) \tag{6}$$

Therefore, the probability that the price of service k equal to PR_k keeps T_{ik} less than τ_k can be obtained as follows:

$$p(T_{ik} \leq \tau_k) = 1 - e^{-\lambda_{ik}\tau_k} \tag{7}$$

Equation (7) is equal to the probability that the reservation price of each customer in cluster i during the time interval τ_k is higher than the price PR_k, or $p\left(R_{ik}^{m_i}(\tau_k) \geq PR_k\right)$. Therefore, equation (7) is the same as equation (5).

It is assumed that the reservation price for each customer in each cluster follows a normal distribution. In this regard, parameters $R_{ik}(\tau_k)$ and $\sigma_{ik}^2(\tau_k)$ are respectively the mean and the variance of the normal distribution, which correspond to the customer's reservation price within cluster i for service k. So, $R_{ik}^{m_i}(\tau_k) \sim N\left(R_{ik}(\tau_k), \sigma_{ik}^2(\tau_k)\right)$. The standard normal distribution of reservation price for each customer of cluster i for service k is presented in equation (8):

$$\frac{R_{ik}^{m_i}(\tau_k) - R_{ik}(\tau_k)}{\sigma_{ik}(\tau_k)} = z_{ik}^{m_i}(\tau_k) \tag{8}$$

Therefore, the value of $p\left(R_{ik}^{m_i}(\tau_k) \geq PR_k\right)$ can be calculated as follows:

$$p\left(R_{ik}^{m_i}(\tau_k) \geq PR_k\right) = p\left(z_{ik}^{m_i}(\tau_k) \geq \frac{PR_k - R_{ik}(\tau_k)}{\sigma_{ik}(\tau_k)}\right) = 1 - \varphi\left(\frac{PR_k - R_{ik}(\tau_k)}{\sigma_{ik}(\tau_k)}\right), \tag{9}$$

where φ represents the standard normal cumulative distribution function. If the right side of equation (5) is set equal to that of equation (9), we have:

$$1 - e^{-\lambda_{ik}\tau_k} = 1 - \varphi\left(\frac{PR_k - R_{ik}(\tau_k)}{\sigma_{ik}(\tau_k)}\right) \Longrightarrow \frac{PR_k - R_{ik}(\tau_k)}{\sigma_{ik}(\tau_k)} = \varphi^{-1}\left(e^{-\lambda_{ik}\tau_k}\right) \tag{10}$$

Based on equation (10) and concerning non-negative customers' reservation prices, the average customer's reservation price of cluster i for service k when the average time interval between two consecutive uses (purchases) of this service is at least τ_k, is equal to:

$$R_{ik}(\tau_k) = max\left\{PR_k - \sigma_{ik}(\tau_k).\varphi^{-1}\left(e^{-\lambda_{ik}\tau_k}\right), 0\right\} \quad \forall i = 1, \dots, I \;,\; k = 1, \dots, K \tag{11}$$

Therefore, in each cluster and for each proposed price, the relationship between the mean and the variance of reservation price subject to the time interval between two purchases can be expressed by equation (11). This relationship for different groups of consumers with various tastes can be justified. For example, if in a given cluster during the *TI* period, none of the customers show a tendency to use service k (assuming the standard deviation of the customer's reservation price of cluster i is the same for all customers), then λ_{ik} is very small and tends to be zero. Thus, $\varphi^{-1}\left(e^{-\lambda_{ik}\tau_k}\right)$ will approach infinity and consequently $R_{ik}(\tau_k)$ becomes zero. In contrast, if in this cluster, the customers' willingness to use service k is increased until λ_{ik} is more than $\frac{-\ln(\varphi(0))}{\tau_k}$, the customer's reservation price for the service k will be more than the current price, *i.e.*, $R_{ik}(\tau_k) \geq PR_k$. In this situation, if the mobile service operator offers a price more than the current price (PR_k) and less than the customer's reservation price $\left(R_{ik}(\tau_k)\right)$, the customer's willingness to buy would not be less than the current situation.

The above definition of the customer's reservation price not only covers customer willingness to use a service but also is directly relevant to their needs, income levels, and anything that would affect customer demand. So, it is logical to consider the distribution of customers' reservation prices for a service proportional to its demand distribution. For example, it can be expected that in case the demand diversity of a specific service in a given cluster is high, customers of that cluster would have the same diversity of willingness to use the service. This issue can be proved by statistical logic if the customer community follows a normal distribution. It is important to note that the initial assumption of normal distribution of reservation prices is only valid if the reservation price of each customer for each service is a linear combination of his/her demand for that service with a zero y-intercept. For instance, in the case of a quadratic combination, the customer's reservation price would follow a chi-square distribution. Moreover, when a customer does not demand a service, his/her reservation price would be zero; so, the y-intercept would be equal to zero. In this paper, we assume that the customer demand of cluster i for service k follows a normal distribution with parameters μ_{ik} and σ_{ik}. Now, the standard deviation of the customer's reservation price $\left(\sigma_{ik}(\tau_k)\right)$ can be obtained by calculating the coefficient of variation $(C.V)$ for each cluster. In each community, $C.V$ is the ratio of standard deviation to mean, and it is useful for comparing two free scale variables. For customers of cluster i, $C.V$ of reservation price for service k provided to τ_k can be calculated as follows:

$$C.V = \frac{\sigma_{ik}(\tau_k)}{PR_k - \sigma_{ik}(\tau_k).\varphi^{-1}(e^{-\lambda_{ik}\tau_k})} \tag{12}$$

Moreover, the demand coefficient of variation for services k by customers of cluster i in the price level PR_k is equal to ${}^{\sigma_{ik}}/_{\mu_{ik}}$. Since the customer demand distribution is proportional to the customer's reservation price distribution, the right side of equation (12) is equal to ${}^{\sigma_{ik}}/_{\mu_{ik}}$. So, $\sigma_{ik}(\tau_k)$ will be obtained as the following form:

$$\sigma_{ik}(\tau_k) = \frac{\sigma_{ik}.PR_k}{\mu_{ik} + \sigma_{ik}.\varphi^{-1}(e^{-\lambda_{ik}\tau_k})}, \forall i = 1,\dots,I,\ \ \forall k = 1,\dots,K \tag{13}$$

By replacing equation (13) in equation (11), $R_{ik}(\tau_k)$ will be calculated as follows:

$$\begin{aligned} R_{ik}(\tau_k) &= max\left\{PR_k\left[1 - \frac{\sigma_{ik}.\varphi^{-1}\left(e^{-\lambda_{ik}\tau_k}\right)}{\mu_{ik} + \sigma_{ik}.\varphi^{-1}(e^{-\lambda_{ik}\tau_k})}\right], 0\right\} \\ &= max\left\{PR_k\left[\frac{\mu_{ik}}{\mu_{ik} + \sigma_{ik}.\varphi^{-1}(e^{-\lambda_{ik}\tau_k})}\right], 0\right\}, \forall i = 1,\dots,I, \forall k \\ &= 1,\dots,K \end{aligned} \tag{14}$$

where $\frac{\mu_{ik}}{\mu_{ik}+\sigma_{ik}.\varphi^{-1}\left(e^{-\lambda_{ik}\tau_k}\right)}$ is the reservation price coefficient provided to τ_k for cluster i. It can be observed that $R_{ik}(\tau_k)$ increases as τ_k increases. This means that higher prices of service are acceptable for customers when the time interval between two purchases increases (i.e., there are fewer purchases in a given time interval). By calculating $R_{ik}(\tau_k)$, we can obtain the reservation price of an offered bundle.

The proposed services are considered to have independent values and are not supplements or substitutes for each other. Then, the reservation price for a bundle can be defined as follows:

$$RE_i = \sum_{k=1}^{K} n_{ik}\, R_{ik}(\tau_k),\ \ \forall i = 1,\dots,I \tag{15}$$

where, n_{ik} is the number of service k in the offered bundle to cluster i. So, the time interval between two successive purchases can be obtained as below:

$$\tau_k = \frac{TI}{n_{ik}},\ \ \forall i = 1,\dots,I,\ \ \forall k = 1,\dots,K \tag{16}$$

By substituting equations (14) and (16) into equation (15), the customer's reservation price of cluster *i* for the offered bundle to this cluster is obtained as follows:

$$RE_i = \sum_{k=1}^{K} n_{ik}\, PR_k \left[\frac{\mu_{ik}}{\mu_{ik} + \sigma_{ik}.\varphi^{-1}\left(e^{-\lambda_{ik}\frac{TI}{n_{ik}}}\right)} \right], \quad \forall i = 1,\dots,I \tag{17}$$

The above equation is used as an equality constraint in the optimization problem that follows (i.e., Appendix D).

Appendix D – Optimization Problem

Bundles' contents and prices are optimized in such a way that total revenue and/or total consumer surplus (the two main objectives of the mobile service operator) are maximized. As discussed in the text, we consider bundle composition, encompassing both the types of services included and the number of services offered.

It should be evident that the primary goal of any business is profitability. For information items with low marginal cost, the amount of profit is approximately equal to income. Therefore, focusing on either will lead to similar results. Equation (18) presents the total revenue of mobile service operators over the billing period *TI*.

$$\begin{aligned} Total\ revenue &= \sum_{i}^{I}\sum_{m_i=1}^{N_i}(p_i - C_i)X_i^{m_i} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{m_i=1}^{N_i} PR_k\left(U_{ik}^{m_i} - n_{ik}\right)H_{ik}^{m_i}X_i^{m_i} \\ &+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{m_i=1}^{N_i} PR_k U_{ik}^{m_i}\left(1 - X_i^{m_i}\right) \end{aligned} \tag{18}$$

In the above equation p_i and C_i are the price and cost of the offered bundle to cluster *i*. The binary variable $X_i^{m_i}$ is one if the customer m_i of cluster *i* purchases the offered bundle, which implies that $RE_i > p_i$. The Binary variable $H_{ik}^{m_i}$ will be zero if the number of service *k* is greater than the demand for it, i.e., $n_{ik} > U_{ik}^{m_i}$), otherwise, it will be one. Equation (18) is composed of three terms. The first term is the amount of income obtained from customers who are willing to buy the proposed bundles. In some cases, the amount of customer demand/consumption may be more than what is offered to them ($i.e. U_{ik}^{m_i} > n_{ik}$); so, they satisfy their surplus needs through individual purchases, where the revenue associated with this type of selling is captured by the second term. Finally, the third term is the amount of revenue associated with selling services to customers who prefer individual buying; for such customers, buying a bundle would either not lead to a surplus or probably cause a value loss.

In addition to maximizing revenue through bundle selling, other objectives can be important for a firm. One of these objectives is maximizing consumer surplus in all or some clusters. Consumer surplus can be defined as the difference between a customer's reservation price for an offered bundle to their cluster and the set price of the bundle.

$$S_i = \max\left(RE_i - p_i, 0\right) \qquad ,\forall i = 1,\dots,I \tag{19}$$

According to equation (19), customers in each cluster can be divided into two groups. The first group (I) includes customers whose reservation price for the proposed bundle is higher than the optimal price set by the firm. Thus, these customers will attempt to buy the bundle to attain its surplus. However, the second group (II) includes those customers whose perceived value for the bundle is less than the price set by the firm; these customers have no willingness to buy the offered bundle and would prefer individual buying instead.

Equation (20) shows the total surplus of the community which is a weighted sum of the consumer surplus over all clusters. The weight of each cluster can be assigned based on the importance that the mobile service operator considers for its customer segments.

$$Total\ surplus = \sum_{i=1}^{I} w_i S_i \tag{20}$$

According to equations (18) and (20), the most important objectives from the mobile service operator's perspective are maximizing total revenue and maximizing total consumer surplus. This calls for a bi-objective optimization problem to evaluate decision alternatives. To handle such a bi-objective problem, we use an aggregate objective function. Assume parameters $\gamma_1$and γ_2 show the objective function coefficients. So, the aggregate objective function can be expressed as a linear combination of the abovementioned objectives, as shown in equations (21) & (22):

$$max\, \gamma_1(Total\ revenue) + \gamma_2(Total\ surplus) \tag{21}$$

Variables $\gamma_1$and γ_2 are determined by the managers of the mobile service operator, where γ_1+$\gamma_2 = 1$. Now, the bundle optimization problem can be presented as follows:

$$max\, \gamma_1 \left(\sum_{i}^{I} \sum_{m_i=1}^{N_i} (p_i - C_i) X_i^{m_i} + \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{m_i=1}^{N_i} PR_k \left(U_{ik}^{m_i} - n_{ik}\right) H_{ik}^{m_i} X_i^{m_i} + \sum_{i=1}^{I} \sum_{k=1}^{K} \sum_{m_i=1}^{N_i} PR_k U_{ik}^{m_i} \left(1 - X_i^{m_i}\right) \right) + \gamma_2 \left(\sum_{i=1}^{I} w_i S_i \right) \tag{22}$$

$Subject\ to$:

$$RE_i = \sum_{k=1}^{K} n_{ik}\, PR_k \left[\frac{\mu_{ik}}{\mu_{ik} + \sigma_{ik}.\varphi^{-1}\left(e^{-\lambda_{ik}\frac{TI}{n_{ik}}}\right)} \right] \quad \forall\, i = 1, \dots, I \tag{17}$$

$$S_i = \sum_{m_i=1}^{N_i} (RE_i - p_i) X_i^{m_i} \qquad \forall i = 1, \dots, I \tag{23}$$

$$\sum_{k=1}^{K} n_{ik} PR_k - p_i \geq 0 \qquad \forall\, i = 1, \dots, I \tag{24}$$

$$\left(U_{ik}^{m_i} - n_{ik}\right) H_{ik}^{m_i} \geq 0 \qquad \forall\, i = 1, \dots, I \quad , k = 1, \dots, K \quad , m_i = 1, \dots, N_i \tag{25}$$

$$\left(n_{ik} - U_{ik}^{m_i}\right)\left(1 - H_{ik}^{m_i}\right) \geq 0 \qquad \forall\, i = 1, \dots, I \quad , k = 1, \dots, K \quad , m_i = 1, \dots, N_i \tag{26}$$

$$(RE_i - p_i) X_i^{m_i} \geq 0 \qquad \forall\, i = 1, \dots, I \quad , m_i = 1, \dots, N_i \tag{27}$$

$$(p_i - RE_i)\left(\sum_{k=1}^{K} H_{ik}^{m_i}\right)\left(1 - X_i^{m_i}\right) \geq 0 \qquad \forall\, i = 1, \dots, I \quad , m_i = 1, \dots, N_i \tag{28}$$

$$(p_i - RE_i)\left(1 - \sum_{k=1}^{K} H_{ik}^{m_i}\right) X_i^{m_i} \geq 0 \qquad \forall i = 1, \dots, I \quad , m_i = 1, \dots, N_i \tag{29}$$

$$n_{ik} \leq \max_{m_i} U_{ik}^{m_i} \qquad \forall i = 1, \dots, I \quad , k = 1, \dots, K \tag{30}$$

$$p_i - C_i \geq 0 \qquad \forall\, i = 1, \dots, I \tag{31}$$

$p_i = v_1 V_i \qquad \forall\, i = 1,\ldots,I$ (32)

$n_{ik} = v_2 \acute{V}_{ik} \qquad \forall i = 1,\ldots,I \quad ,k = 1,\ldots,K$ (33)

$p_i \geq 0 \qquad \forall i = 1,\ldots,I$ (34)

$RE_i \geq 0 \qquad \forall i = 1,\ldots,I$ (35)

$S_i \geq 0 \qquad \forall i = 1,\ldots,I$ (36)

$n_{ik} \geq 0: integer \qquad \forall i = 1,\ldots,I \quad , k = 1,\ldots,K$ (37)

$V_i \geq 0: integer \qquad \forall i = 1,\ldots,I$ (38)

$\acute{V}_{ik} \geq 0: integer \qquad \forall i = 1,\ldots,I \quad ,k = 1,\ldots,K$ (39)

$X_i^{m_i} \in \{0,1\} \qquad \forall\, i = 1,\ldots,I \quad ,m_i = 1,\ldots,N_i$ (40)

$H_{ik}^{m_i} \in \{0,1\} \qquad \forall i = 1,\ldots,I, k = 1,\ldots,K \quad ,m_i = 1,\ldots,N_i$ (41)

The objective function is a linear combination of the total revenue of the mobile service operator and the total consumer surplus. Constraint (17) describes the customer's reservation price in each cluster for the offered bundle (obtained in Appendix C). Constraint (23) calculates the total consumer surplus of clusters i, in the case of buying the offered bundle. According to constraint (24), the price of each bundle must be less than or equal to the sum of its service prices multiplied by their volume; this constraint is needed to encourage customers to buy offered bundles by considering discounts in the bundle selling strategy. Constraints (25) and (26) are designed to properly initialize $H_{ik}^{m_i}$, and to ensure the adequacy of services within each bundle for satisfying customer demands; in this regard, If the number of service k is greater than the demand for it ($i.e.\ n_{ik} > U_{ik}^{m_i}$), then $H_{ik}^{m_i}$ will be zero, otherwise, it will be one. Constraints (27)-(29) are used to initialize binary variable $X_i^{m_i}$, which is one if the customer m_i of cluster i purchases the offered bundle ($i.e. RE_i > p_i$). Based on constraint (27), a consumer might purchase a bundle that would lead to a surplus for him/her, and otherwise, the bundle will not be purchased. According to constraint (28), a customer buys a bundle if it leads to surplus and his/her need for at least one service type is higher than the bundle capacity. The constraint (29) indicates that if all services of a bundle exceed than customer's demand, he/she will not show a tendency to buy the bundle. Constraint (30) shows the upper bound for the number of services in each bundle; it is equal to the highest demand for services k in cluster i. Constraint (31) implies that for each bundle, the offered price must be greater than the cost of the bundle. Constraints (32) and (33) perform price rounding for offered bundles; for this purpose, two integer variables V_i and $\acute{V}_{ik}$ are introduced to round the price of the offered bundle for each cluster and the number of service k for the offered bundle to cluster i, respectively. Constraints (34)-(41) show the ranges for the model decision variables.

The above formulation is a Mixed Integer Nonlinear Programming (MINLP) model. Generally, in such models, there are numerous local optima, which make it difficult to find the exact solution. Therefore, applying common informed and uninformed search techniques such as branch and bound and cutting plane would be inefficient. Moreover, most of these techniques are very time-consuming. Hence, approaches that could obtain high-quality solutions at a reasonable time would be most useful. In this regard, metaheuristic algorithms have been successfully used to obtain near-optimal solutions for MINLP models. As mentioned in the text, a simulated annealing (SA) algorithm is used to solve the problem.